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Optimal second harmonic generation

Q: What is the optimal focusing condition in second harmonic generation?

A:

A second harmonic generation (SHG) is a common nonlinear process. From couple wave theory, the following equations can be derived for further calculation:

$$\frac{dE_\omega}{dz} \propto E_{2\omega} E_\omega^* e^{-j\Delta kz}$$

$$\frac{dE_{2\omega}}{dz} \propto E_\omega^2 e^{j\Delta kz}$$

However, the couple wave equations are derived under the assumption of plane wave approximation, but the SHG conversion efficiency is depends on optical intensity and higher conversion efficiency can be obtained with a focused beam.

For a Gaussian beam, the electric field is rewritten into

$$E = E_0(z) \cdot u(x, y, z) e^{\omega t - kz}$$

$$u(x, y, z) = \frac{w_0}{w(z)} e^{-\frac{x^2+y^2}{W^2(z)}} e^{-jk\frac{x^2+y^2}{2R(z)}} e^{j\phi(z)}$$

where the nonlinear polarization can be further modified by

$$P_{NL}^{(2\omega)} \propto E_\omega(z)^2 u_\omega^2(x, y, z) e^{-2jk_\omega z}.$$

Under the non-depleted pump condition, the relation between second harmonic field and nonlinear polarization is

$$u_{2\omega} \frac{\partial E_{2\omega}}{\partial z} \propto P_{NL}^{2\omega} e^{jk_{2\omega} z}$$

$$\rightarrow \frac{\partial E_{2\omega}}{\partial z} \propto E_\omega(0)^2 \frac{u_\omega^2(x, y, z)}{u_{2\omega}(x, y, z)} e^{j\Delta kz}$$

Since the second-harmonic beam has a factor of $\sqrt{2}$ smaller beam radius than the fundamental beam, the fiber propagation equation is reduced into

$$\frac{\partial E_{2\omega}}{\partial z} \propto E_\omega(0)^2 \frac{W_0}{W(z)} e^{j\Delta kz} e^{j\phi_\omega(z)},$$

and $\frac{e^{j\phi_\omega(z)}}{W(z)}$ is the extra term compared to the plane wave assumption.

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In order to find the optimal focusing condition, we integrated the equation numerically (assume focus at the center of the crystal), which will be

$$\alpha = \int_{-L/2}^{L/2} \frac{e^{j\tan^{-1}\left(\frac{z}{z_0}\right)} e^{j\Delta k z}}{\sqrt{1+\frac{z^2}{z_0^2}}} dz$$

The equation of second harmonic intensity therefore changes:

$$I_{2\omega} = \Gamma^2 I_\omega^2 L^2 \rightarrow I_{2\omega} = \Gamma^2 I_\omega^2 \alpha^2,$$

To achieve the highest conversion efficiency, the wave vector mismatch Δk should compensate of the linear fit of the gouy phase shift $\tan^{-1}\left(\frac{z}{z_0}\right)$.

The output power will be the integration of output intensity in space,

$$\rightarrow P_{2\omega} = I_{2\omega}\int_0^\infty 2\pi\rho|u_{2\omega}|^2 d\rho = \Gamma^2 I_\omega{}^2 \alpha^2 \int_0^\infty 2\pi\rho|u_{2\omega}|^2 d\rho = \Gamma^2\alpha^2 P_\omega{}^2 \frac{\int_0^\infty 2\pi\rho|u_{2\omega}|^2 d\rho}{\left|\int_0^\infty 2\pi\rho|u_\omega|^2 d\rho\right|^2} =$$

$$\Gamma^2\alpha^2 P_\omega{}^2 \frac{1}{\pi W_0{}^2} = \gamma P_\omega{}^2 \frac{\alpha^2}{b} = \gamma P_\omega^2 L h, \qquad h = \frac{\alpha^2}{Lb}$$

✧ Optimal Conditions:

h = 1.068 at *L*/*b* = 2.84 (Boyd, et.al, 1968)

HCP simulation: *h* = 1.066 at *L*/*b* = 2.81

Reference:

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