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Q: How does the phase-matching bandwidth affect the SHG of ultrashort pulse?

A:

Phase-matching bandwidth:

It is well known that there are two conditions need to be satisfied for a nonlinear three wave mixing process:

$$\omega_3 = \omega_2 + \omega_1 \qquad (1)$$

$$k_{\omega_3} = k_{\omega_2} + k_{\omega_1} \qquad (2)$$

Equation (1) stands for the energy conservation, the sum of the input and the output photon energy will be the same. Equation (2) stands for the momentum conservation that the sum of the input wave vector should be equal to the output wave vector and so called phase-matching. The phase matching bandwidth therefore could be calculated according to the above equation. In this article we take SHG as the example which phase-matching condition is shown below:

$$\Delta k = 2k_{\omega} - k_{2\omega} = 0$$

The phase-mismatch could be calculated by applying the Taylor expansion to the above equation:

$$\Delta k(\Delta\omega) = 2k_{\omega_1+\Delta\omega} - k_{2\omega+\Delta 2\omega} \approx 2k_{\omega_1} - k_{2\omega} + 2\Delta\omega\frac{dk_{\omega}}{d\omega} - 2\Delta\omega\frac{dk_{2\omega}}{d\omega}$$

To calculate the bandwidth in full width half maximum (FWHM) which implies Δk ~ 2.78 $/L$. Assuming the zero-order phase mismatch will be compensated by artificial wave vector to have $2k_{\omega 1}$- $k_{\omega 2}$ -Δk = 0, the residual 1st order term will determine the acceptance phase-matching bandwidth. In fact, the 1st order term implies the difference of group velocity of the involved waves. In a pulsed SHG, the discrepancy of group velocity between fundamental and second-harmonic waves reveals the needed bandwidth of phase-matching and also the temporal walk-off.

One can rewrite the equation for the FWHM bandwidth as below

$$2\Delta\omega\left(\frac{1}{V_{g,\omega_1}} - \frac{1}{V_{g,\omega_2}}\right) \sim \frac{2.78}{L}$$

$$FWHM = 2\Delta\omega = \frac{2.78}{L\left(1/V_{g,\omega} - 1/V_{g,2\omega}\right)}$$

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Above formula suggests some general concepts for the SHG bandwidth:

1. The bandwidth is inverse proportional to the crystal length.
2. The bandwidth is strongly depends on the mismatch of the group velocity of the fundamental and second harmonic wavelength.

SHG of ultrashort pulse:

For an ultrashort pulse laser, the bandwidth of the laser is quite large, which typically can be estimated by:

$$\Delta t \Delta \omega \sim \frac{0.443}{2\pi}$$

Since the term $L/(1/V_{g1} - 1/V_{g2})$ equivalent to the temporal walk-off of passing through the crystal, the relation between SHG bandwidth and the walk-off therefore can be derived as the function below:

$$\Delta \tau_{walk-off} \sim \frac{2.78}{FWHM}$$

One can compare the crystal walk-off time with the laser pulse width or bandwidth to have some thought to the SHG result. Here we conclude some useful rules:

1. If the crystal walk-off time << laser pulse width, the SHG pulse width will be smaller than the input pulse.
2. If the crystal walk-off time > laser pulse width, the SHG pulse width will be similar to the crystal temporal walk-off, and the SHG pulse shape in time-domain will be similar to a squared pulse.

Below is a reference figure for the walk-off under different SHG input wavelength for PPLN.

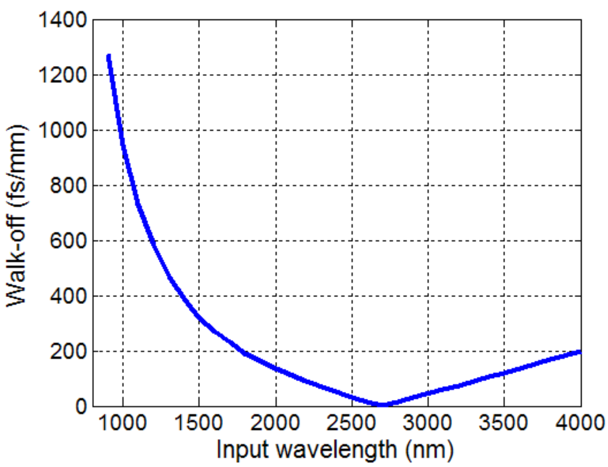

Fig. 1 Temporal Walk-off for the PPLN between fundamental and SHG wavelength.

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Reference:

[1] Andrew Weiner, "*Ultrafast Optics*", Wiley Publishing, 2009, Chapter 4